Lect. 3: Light Propagation in Various Media

Characteristics of media: $\begin{cases} \varepsilon : \text{permittivity} \\ \mu : \text{permeability} \end{cases}$ $\begin{cases} \overline{D} = \varepsilon \overline{E} \\ \overline{B} = \mu \overline{H} \end{cases}$

- Assume $\mu = \mu_0$ in this course.

- With different ε, how do plane-wave solutions chnage?

For example, $\overline{E} = \overline{x} E_0 e^{j(\omega t - kz)}$

$k = \omega\sqrt{\mu\varepsilon} \quad = nk_0 \quad (k_0 = \omega\sqrt{\mu\varepsilon_0}, \; n = \sqrt{\varepsilon / \varepsilon_0}$; refractive index)

➔ Dielectric media

=> changes in λ phase velocisty ($v_p = \dfrac{\omega}{k}$) group velocity ($v_g = \dfrac{\partial \omega}{\partial k}$)

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Medium with conductivity $\sigma \neq 0 \quad \overline{J} = \sigma \cdot \overline{E} \neq 0$

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$$

$$\nabla \bullet \overline{D} = \rho$$

$$\nabla \bullet \overline{B} = 0$$

With plane-wave solutions

$$\nabla \times \overline{E} = -j\omega \overline{B}$$

$$\nabla \times \overline{H} = \overline{J} + j\omega \overline{D}$$

$$\nabla \bullet \overline{D} = \rho$$

$$\nabla \bullet \overline{B} = 0$$

(No t-dependence)

$$\nabla \times \overline{H} = \sigma \overline{E} + j\omega\varepsilon \overline{E}$$

$$= j\omega(\varepsilon - j\frac{\sigma}{\omega})\overline{E}$$

$$= j\omega\varepsilon_c \overline{E}$$

$$\varepsilon_c \equiv \varepsilon - j\frac{\sigma}{\omega} = \varepsilon' - j\varepsilon'', \ \varepsilon'' = \frac{\sigma}{\omega}$$

➔ Lossy medium has complex ε

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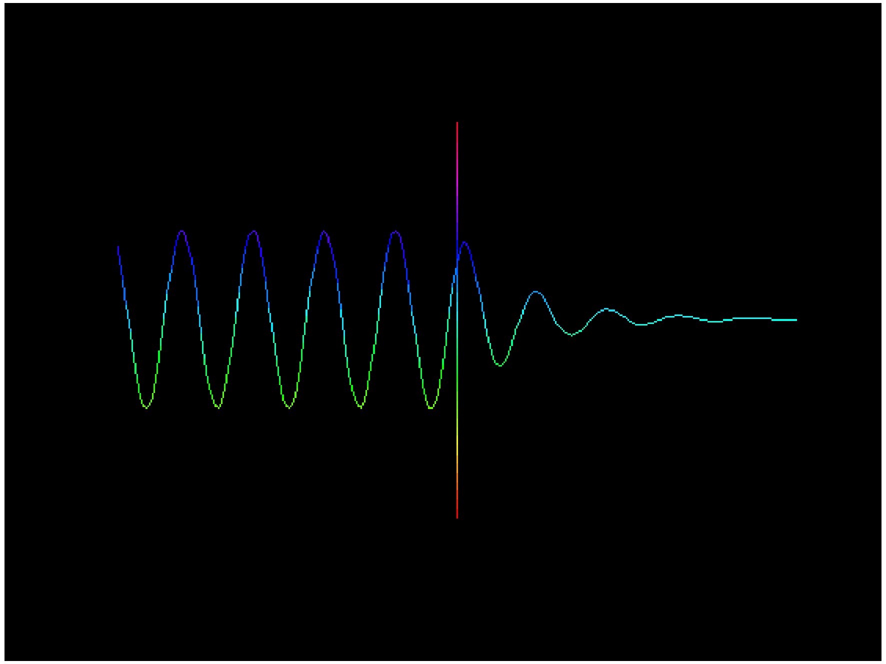
With complex ε

$k = \omega\sqrt{\mu\varepsilon}$ $\quad$ k is also complex! $\quad$ $k = \beta - j\alpha$

Consider $\overline{E} = \overline{x}E_0 e^{-jkz}$ ($e^{j\omega t}$ is often emitted)

$$= \overline{x}E_0 e^{-j(\beta - j\alpha)z} = \overline{x}E_0 e^{-j\beta z} e^{-\alpha z}$$



EM waves get attenuated in conductive medium!

➔ lossy medium

In general,
medium properties are reflected in ε and k

➔ Influence wave propagation

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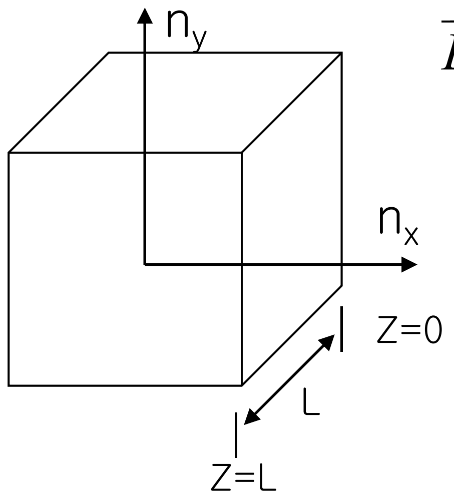
Birefringent medium:

Reflective index n depends on E-field direction

E_x experiences n_x and E_y experiences n_y



$\vec{E}_{in} = (\bar{x}+\bar{y}) E_0 e^{-jk_0 z}$ enters birefringent material at z=0

Polarization of E_{out} at z=L ?

$$\bar{E}_{\text{out}}(z=L) = \bar{x}E_0 e^{-jn_x k_0 L} + \bar{y}E_0 e^{-jn_y k_0 L}$$

$$\frac{E_y}{E_x} = e^{-j(n_x - n_y)k_0 L}$$

$(n_x - n_y)k_0 L = m\pi$: linear polarization

$= \frac{(2m+1)}{2}\pi$: circular polarization

otherwise: elliptical polarization

Birefringet medium can change polarization of EM waves!

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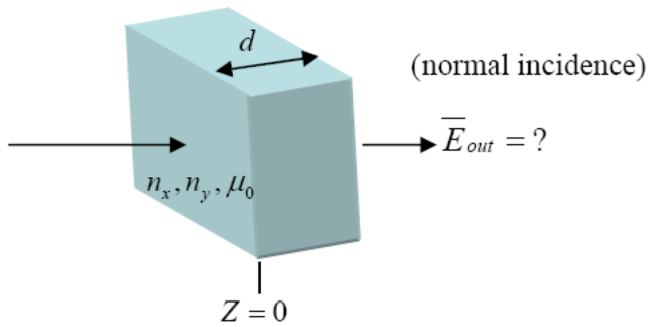
Homework: Due on 9/13 before Tutorial

An EM wave is incident on a certain material as shown below.

μ_0, ε_0

$\overline{E}_{in} = E_0(\overline{x} + \overline{y})e^{-jk_0 z_0}$



(a) What is the polarization of the input EM wave?

(b) The refractive index of the material depends on the direction of E-field: it is n_x when E-field is in x-direction and n_y when y-direction. Note that $n_x \rangle n_y$.

Assuming there is no reflection at both input and output interface, determine the output EM wave. Express your answer in terms of parameters given in above figure.

(c) It is possible to change the polarization of the input wave by controlling the material thickness. Determine the smallest possible material thickness so that output EM wave has left-hand circular polarization.